

Measurement of Radio-Frequency Radiation Pressure: The Quest for a NEW SI Traceable Power Measurement

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Abstract—In the quest for a new SI traceable power measurement, we performed measurements of the radiation pressure of a radio-frequency (RF) electromagnetic field. There are several groups around the world investigating methods to perform more direct SI traceable measurements of RF power (where RF is defined to range from 100s of MHz to THz). A measurement of radiation pressure offers the possibility for a power measurement traceable to the kilogram and to Planck's constant through the redefined SI. Towards this goal, we demonstrate the ability to measure the radiation pressure/force carried in a field at 2.45 GHz and 15 GHz using two different devices.

Keywords—power measurements, radiation pressure, radiation force, new SI metrology

I. INTRODUCTION

One of the keys to developing new science and technologies is to have sound metrology tools and techniques. Fundamental to all electromagnetic (EM) and electromagnetic compatibility (EMC) measurements is having accurately calibrated probes, antennas, and power meters in order to measure either electric (E) fields or power. A stated goal of international metrology organizations, including the National Institute of Standards and Technology (NIST), is to make all measurements directly traceable to the International System of Units (SI). The current method of power traceability is typically based on an indirect path through a thermal measurement using a calorimeter, in which temperature rise created by absorbed microwave energy is compared to the DC electrical power. This is traceable to quantum voltage, Hall resistance, and electronic charge.

The world of measurement science is changing rapidly with the SI redefinition planned for late 2018. As a result of the shift towards fundamental physical constants, the role of primary standards must change. This includes power, which is currently traceable to electrical units through thermal detectors. Various groups are investigating different approaches to perform more direct SI traceable power measurements. These include measurements based on Rydberg atoms [1]–[3] and measurement of power through the radiation pressure carried by an electromagnetic (EM) field [4–5]. The Rydberg atom approach allows a direct SI traceable

measurement through Planck's constant, while the radiation pressure approach allows for a SI traceable measurement through the kilogram. However, with the redefinition of the SI in late 2018, the kilogram will be directly traceable to Planck's constant [6].

Radiation pressure from laser power has been measured under a variety of conditions. However, at radio frequencies, we know of only one measurement [7] performed at a power level of ten's of watts. However, that measurement was carried out with a torsion style balance which limits operation in practical measurement conditions, and precludes traceability to the kilogram through direct weighing of a calibrated mass. The concept of measuring radiation pressure stems from the fact that EM fields carry a momentum as they propagate through space and this momentum results in an EM pressure expressed as (in units of N/m^2) [8]

$$\text{Pressure} = \frac{\langle \mathbf{E} \times \mathbf{H} \rangle}{c}, \quad (1)$$

where the symbol “ $\langle \rangle$ ” represents time average, c is the speed of light *in vacuo* (in units of m/s), and \mathbf{E} (in units of V/m) and \mathbf{H} (in units of A/m) are the electric and magnetic fields. By measuring this pressure (via a force measurement), we can obtain a measurement of the RF power carried in the RF field. From this, it can be shown that when the EM field is normally incident on a device to measure force with a reflecting surface (i.e., a scale) the radiation force is given by (in units of N):

$$F = \frac{2P}{c} \left(R + (1-R) \frac{\alpha}{2} \right), \quad (2)$$

where P (in units of W) is the RF power incident on the scale reflector, c is the speed of light *in vacuo* (in units of m/s), R is the reflectance of the surface, and α is the fraction of non-reflected RF that is absorbed. In this experiment, we assume all of the non-reflected RF energy is absorbed, or $\alpha = 1$. The factor of “2” in front of P is a result from the conservation of linear momentum.

Here we will discuss two sets of experiments with two different devices used to perform RF radiation pressure measurements at 2.45 GHz and 15 GHz. The first device allows for power measurements in the range of 2 W -to- 50 W, and the second device allows for measurements above 100 W.

II. RADIATION PRESSURE MEASUREMENTS

We used two different force measurement devices. The first device is a parallel-plate capacitor force sensor and the second device is a commercially available mass scale.

A. Parallel-Plate Capacitor Force Sensor

These experiments were performed at 15 GHz. A photo of the experimental setup is shown in Fig 1 and a block diagram in shown Fig. 2. A signal generator (SG) feeds a 200 W amplifier with a WR62 waveguide output. The output was run through a filter and two isolators and then to a waveguide/coax adapter. A 0.5 m cable was used to connect to a second waveguide/coax adapter to feed a second waveguide section (the cable was used to isolate the pressure sensor from mechanical vibrations caused by the amplifier). The waveguide was connected to a directional coupler and then to the last section of waveguide (the “black” waveguide in the figure). The pressure sensor was connected to the output of this black section of waveguide.

The radiation pressure device (shown in Fig. 3) is a capacitor-based force sensor. Upon reflection of the plane-wave RF beam normally incident on an aluminum reflector, a force given by the change in momentum of the reflected beam deflects a silicon spring. This changes the plate spacing of a parallel-plate capacitor, which sets a Wien capacitor bridge out of balance producing a voltage signal. In these experiments, we drove the bridge with a 20 kHz, 1 V sinusoidal source and demodulated the bridge signal with a lock-in amplifier locked to 20 kHz using a 30 ms, 6 dB lowpass filter for noise suppression. We recorded the output of the lock-in amplifier with an oscilloscope set to AC and triggered to the modulation of the RF source.

The spring itself is cut from crystalline silicon wafer. It includes a flat central disk, 20~mm in diameter, which supports the aluminum reflector on the front side (see Fig. 3) and a gold electrode on the back side. The disk is surrounded with three Archimedean spiral legs that are deep etched through the silicon wafer, connecting the central disk to an annular support for mounting. Given normal deflection of the spring, where the central disk moves along its surface normal, the spring stiffness is approximately 170~N/m. The radiation force on the spring from the reflected RF beam is given by eq. (2).

The spring was clamped to a rigid aluminum back plate with a 30 μ m thick polyimide spacer. With no force acting on the spring, the capacitance between the spring and aluminum back plate was 255 pF. Ferrite beads and a low pass filter were used to isolate the bridge electronics from any potential RF leakage, which added 24 pF of parasitic capacitance to the sensor. In this configuration, the signal sensitivity of the

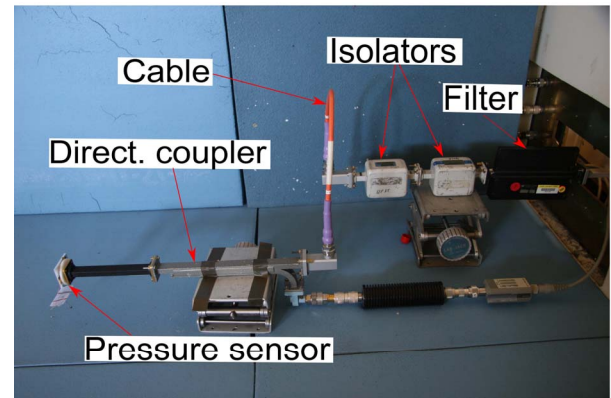


Fig. 1. Experimental setup for 15 GHz radiation pressure measurements.

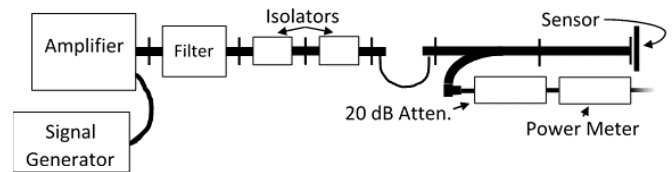


Fig. 2. Block diagram for experimental setup.

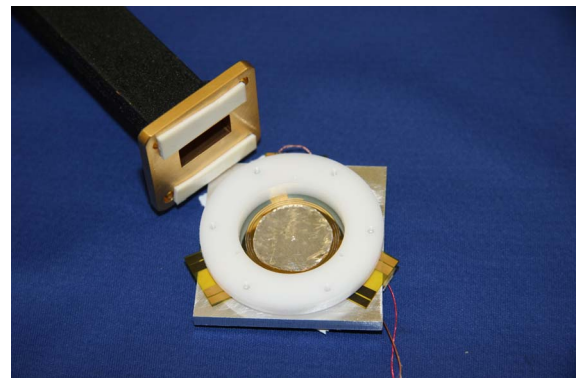


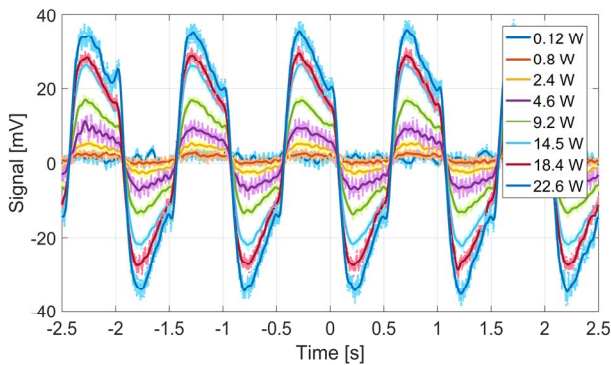
Fig. 3. Photo of the pressure sensor detached from the waveguide.

sensor was approximately -5 V/pF . With a thermocouple attached to the aluminum back plate, we were able to track the sensor temperature throughout RF beam injection. During the experiments, the output of the SG was varied such that the power (measured with a traditional power meter) at the output of the waveguide ranged from 0.12 W to 22.6 W. The RF pressure-sensor measured voltages for these different power levels are shown in Fig 4. Fig. 4(a) shows the signal versus time, where the RF power was modulated at 1 Hz, and Fig. 4(b) shows the signal voltage of the force sensor for different RF power levels. The results in these figures illustrate that the radiated power can be detected with this pressure sensor.

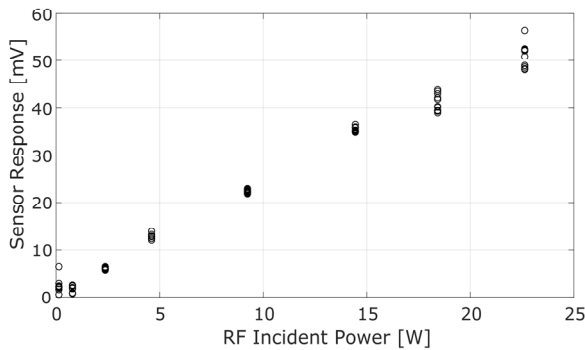
It is instructive to get an estimate of the values of forces the pressure sensor can detect. We do this by using eq. (2) and assuming a reflectance of $R=0.90$, as such eq. (2) reduces to:

$$F = \frac{1.9 P}{c} \quad (3)$$

Fig. 5 shows a plot of this function. For the power range of 1 W to 23 W, we calculated the force range to be 6.3 nN to 145.6 nN. For reference, a grain of maize pollen weighs approximately 2.5 nN; a RDA of vitamin B12 weighs approximately 23.5 nN; a human eyelash weighs approximately 686 nN; and a fruit fly weighs approximately 1960 nN. These various forces are also shown in Fig. 5. Thus, we can detect a force that is equivalent to a grain of maize pollen or 300 times smaller than the gravitational force of a fruit fly.



(a) Signal voltage versus time.



(b) Repeated experiments at various power levels.

Fig. 4: (a) Signal voltage from the pressure sensor, and (b) include repeated experiments.

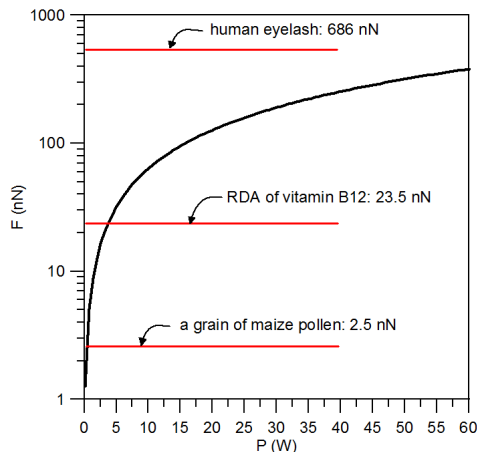


Fig. 5. RF power related to equivalent force (in units of nN). For comparisons, we shown weights of various quantities.

B. Commercially Available Mass Scale

These experiments were performed at 2.45 GHz and we tested the ability to measure the radiation pressure for RF powers on the order of a few hundred Watts. For a source, we used the magnetron from a commercially available microwave oven, which was rated at 1 kW. The magnetron was placed at one end of a WR430 (10.922 cm x 5.461 cm) open-ended rectangular waveguide. The open end of the waveguide was pointed at a commercially available mass scale. This scale is part of device referred to as a radiation pressure power meter (RPPM) designed for measuring laser power levels above 1 kW [9].

Photos of the waveguide source and the experimental setup are shown in Figs. 6 and 7. The experimental setup was placed in a metal enclosure because of the high power used during the experiments.



Fig. 6. Magnetron/waveguide source.



Fig. 7. Waveguide source and RPPM device.

The RPPM is based on a force balance (scale) able to measure a horizontally-directed force with a mass resolution (readability) of ~ 100 nN (about 1/7 the weight of a human eyelash) and a 5 s rise time. An aluminum plate reflector was attached to the force sensing shaft of the scale. The plate had an area of 260 cm^2 , larger than the exit aperture of the waveguide. This plate was placed approximately 5 cm from the exit of the waveguide. With this large-area reflector

attached to the scale, the force readings were particularly susceptible to air currents. To mitigate this, a simple cardboard box was placed around the radiation pressure power meter (see Fig. 8). This helped to reduce the force noise from air currents, but restricted cooling of the scale, causing temperature-dependent drifts of the scale reading with time.

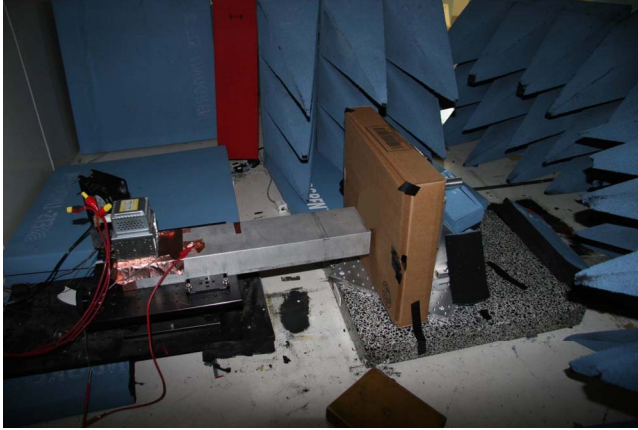


Fig. 8. Box placed around the pressure meter.

When the magnetron was energized, it generated a significant amount of heat, some of which appears to have affected the reading from the RPPM. The force on the RPPM was measured and the power was obtained from eq. (2). The power measured by the RPPM for repeated microwave power injections (4-5 s duration) is plotted in Fig. 9. The exponential rise of the measured power is likely dominated by the RPPM's rise time. The magnetron was not well-cooled in the test configuration and could only be run for 4-5 s due to overheating whereas the RPPM typically requires a 10 s injection for noise reduction. For each injection, the RPPM reported the power as the peak measured power (after correcting for a downward drift due to thermal effects on the force sensor). Multiple measurements yielded the values shown in Fig. 10. These measured powers yielded an average power of about 375 W. As discussed below, this appears to be reasonable given the non-optimal operation without a cooling fan, potential waveguide losses, and some leakage of microwave power around the mirror. The 5 % standard deviation of repeated measurements is likely due to insufficient averaging of the RPPM signal, microwave heating of the force sensor, and possibly variability of the source power.

In order to justify the measured power from the end of the waveguide obtained from the force measurement, we estimated the output power with a heat-capacity measurement. We placed 0.2 L of water at the opening of the waveguide, see Fig. 11. We measured the change in temperature (ΔT) for a given time interval (Δt) with the magnetron turned on. The output power from the waveguide can be approximated by

$$P(W) = 0.997 \frac{kg}{L} \bullet 0.2 L \bullet 4184 \frac{J}{kg C} \bullet \frac{\Delta T}{\Delta t} \quad (4)$$

We performed three sets of experiments and the average power we obtained was 365 W, also shown in Fig. 10. We see that this number is only about 3 % different than the pressure measurements.

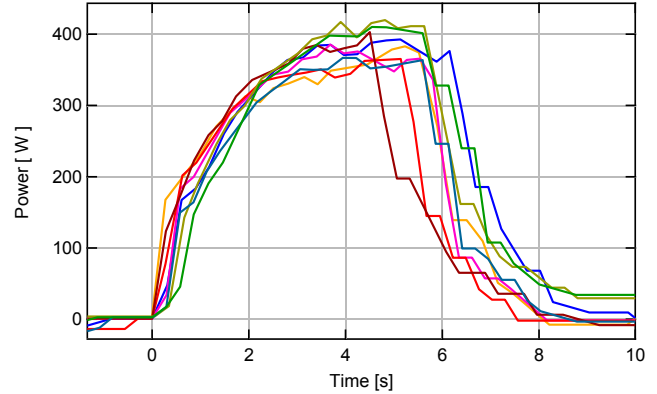


Fig. 9. Waveform from the power measured by the RPPM for eight different experiments.

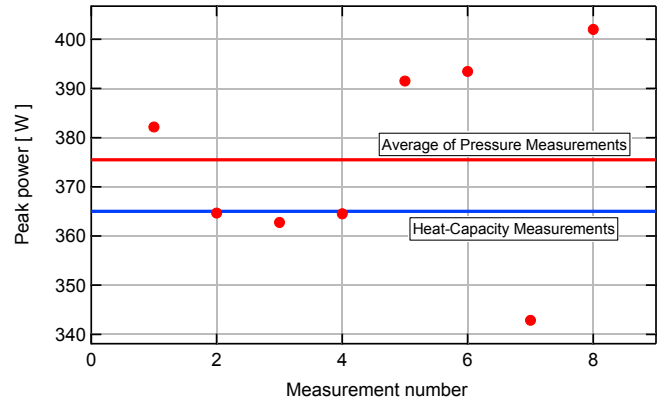


Fig. 10. Multiple measurements for power yielding an average power of about 375 W (the thick line). The dots are the power measurements from the eight data sets from Fig. 9.



Fig. 11. Heat-capacity experiments, water placed at opening of the waveguide.

III. SUMMARY

We have demonstrated that it is possible to detect the radiation pressure carried by an RF field. We illustrated this by performing experiments at 15 GHz and 2.45 GHz using

two different force devices. A parallel-plate capacitor force sensor was used to measure power levels from 2 W to 20 W at 15 GHz. A commercially available mass scale was used to measure power at 375 W at 2.45 GHz. The 2.45 GHz measurements were verified with a separate heat-capacity measurement. These types of power measurements could lead to a fundamentally new approach for calibrating RF power and lead to a new SI traceable approach for RF power. In future work, we will perform similar experiments at different RF frequencies and power levels, and we will calibrate the parallel-plate capacitor force sensor in order to give a direct measurement of the RF power carried in the EM field. Furthermore, this approach has the potential to measure power levels well above kW. However, we hope to develop pressure sensors to measure much smaller power levels (<1 mW).

Understanding the uncertainties of this technique is an important step if this method is to be accepted as a standard calibration approach. The various aspects that contribute to the uncertainties of this technique are currently being investigated. With that said, with the SI redefinition planned for late 2018, this new approach will allow for direct traceable power measurements with significantly improved uncertainties and frequency ranges over current approaches. This technique could potentially allow power measurements and calibrations from 1 mW to 1 MW regardless of frequency (from UV to RF) with one traceability chain.

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